Abstract. In the article some arguments for Hintikka claim from [5], that sentences like: “Some relative of each villager and some relative of each townsman hate each other” have branching logical form not expressible in elementary logic, are considered and then refuted. Attention is focused mainly on referential and inferential arguments proposed in [9].

Key words: Logical Form, Natural Language Semantics, Branching Quantifiers, Computational Complexity.

0. Problem

Hintikka claimed in [5] that sentences like:

(1) Some relative of each villager and some relative of each townsman hate each other.

essentially require nonlinear quantification for expressing their logical form. The logical form of (1) should be writing down as follows:

(2) \forall x \exists y \forall z \exists w [ (V(x) \land T(z)) \Rightarrow (R(x, y) \land R(z, w) \land H(y, w)) ].

The formula (2) is not equivalent to any first order formula (see the Barwise-Kuhnlen Theorem in [1]), but is equivalent to the second order formula:

(3) \exists f \exists g \forall x \forall z [ (V(x) \land T(z)) \Rightarrow (R(x, f(x)) \land R(z, g(z)) \land H(f(x), g(z))) ].

Hintikka’s reading of (1) is called “strong reading”, but we can also assign to (1) “weak reading”, i.e. linear logical form which is expressible in the elementary logic:

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1 This is a draft version of that paper and I would be very grateful for all comments on my e-mail: kszymanik@wp.pl
\[(4) \quad \forall x \forall z \exists y \exists w [(V(x) \land T(z)) \Rightarrow (R(x, y) \land R(z, w) \land H(y, w))].\]

By Hintikka thesis we mean the following,

\[(5) \quad \text{Sentences like (1) have essentially nonlinear logical forms. For example, sentence (1) should be interpreted as (2).}\]

Because of its many philosophical and linguistic consequences Hintikka’s claim has sparked lively controversy (see: [1], [3], [4], [6], [8], [9], and [10]). In this article some arguments presented in the discussion are analysed once again. Our conclusion is that sentences like (1) have logical form expressible in elementary logic, despite what Hintikka stated. In particular we claim that adequate reading for (1) is (4) and not (2). We assumed that the criterion for adequacy of logical form is its agreement with sentence truth conditions. Therefore, we claim that truth conditions of (1) are expressed by (4) and not by (2).

1. Referential argument

The most obvious idea how to choose between proposed readings is to ask a native English speaker about their interpretation of (1) in given situations. One has to show two models of relationship between town and village and ask people whether the sentence (1) is true in each of them. These models are: model for “weak” and for “strong” reading. In general we know that:

\[(6) \quad \forall \exists \phi \Rightarrow \forall \forall \exists \exists \phi \]

Therefore one who accepts both presented situations as adequate has to assign “weak” reading to (1). One agrees to “strong reading”, when accepts only model for “strong” reading. For example, we can present the following models. Dots in the same rectangle are each other’s relatives, everybody is his own relative, and dots connected by line represent villagers and townsmen who hate each other.
People in general interpret (1) as true in the model for “weak reading” (see: [1], [8]), therefore they interpret (1) as a first order sentence.

Nevertheless, Mostowski in [8] claims that native speaker’s inclination toward “weak reading” cannot be accepted as an argument for linear reading of (1) because the problem of recognising the truth value of (2) in finite models is NPTIME-complete (see: Theorem 2.1 in [9]). Assuming Church Thesis in its psychological version, Edmond’s Thesis [2], and P≠NP, it is claimed in [9] that our mind is not equipped with any mechanisms of recognising NP-complete problems. In other words, we cannot perform algorithm for checking the truth value of (2) in presented diagrams by means of our “hardware” in a single step. The authors of [9] conclude that this reasoning refutes an argument from native speaker’s intuition.

We cannot agree with authors’ conclusion. Presented arguments make us believe (under stated assumptions) that our minds are not equipped in any mechanism, allowing us to recognise the truth value of “strong reading” in finite models. But it is not an argument against language competence but rather theoretical base for “empirical result”. If it is true that “model-checking” for the “strong reading” of (1) is beyond our minds ability, then it means that (1) must be understood and interpreted by speakers linearly (of course under authors’ rigorous assumptions, we can still recognise the truth of (4) in given model because “model-checking” for first-order logic is in LOGSPACE).
2. Inferential argument

We can gain knowledge about logical interpretation of sentences not only by analysing referential meanings which are assigned to these sentences by native speakers. The other way is to analyse the inferential dependences of these sentences. The problem of the adequate reading of (1) can be reduced to the problem of logical form of some “easier” sentence which is implied by (1).

If we assume that:

\[(7) \quad \text{Mark is a villager.}\]

we agree that (1) implies:

\[(8) \quad \text{Some relative of Mark and some relative of each townsman hate each other.}\]

If we interpret (1) by its “weak reading” then we must agree that (8) is true in the following diagram:

![Diagram](image)

In [8] it is claimed that this is a dubious consequence of assigning linear interpretation to (1). Mostowski’s reasoning is implicit based on the interpreting (8) as:
(9) \( \exists x \, [R(Mark, x) \land \forall y (T(y) \Rightarrow \exists z (R(y, z) \land H(x, z)))] \).

This is false in the above model. (9) is not implied by “weak reading” of (1), but it is implied by “strong reading”. Therefore Mostowski claims in [8] that (1) must have a branching form not expressible in elementary logic. This argument may be not accepted. Firstly, (8) intuitively seems to be true in the model presented above. Secondly, as logical form of (8) we would rather take:

(10) \( \forall y \, [T(y) \Rightarrow \exists x (R(Jan, x) \land \exists z (R(y, z) \land H(x, z)))] \),

(this is true in the above diagram and followed from the “weak reading” of (1)). (9) claims that there exists such a relative of Mark that he hates some relative of every townsman whereas (8) shows that every townsman has a relative who hates some relative of Mark which is reciprocated. Therefore we can assign (10) as logical form of (8) and hold linear interpretation of (1) without any doubtful consequences.

3. Conclusions

We discussed and refuted two of arguments for Hintikka’s Thesis. In [1] some other arguments against (5) can be found. We claim that the discussion around Hintikka’s Thesis determines us to admit that (1) has a logical form expressible in elementary logic.

Nevertheless, we agree that there are English sentences whose logical form to be expressed needs branching quantifiers. But there are none similar to (1); for their formulating we need different determiners rather than those expressible in elementary logic. Good example of such nonlinear sentence is:

(10) Most villagers and most townsman hate each other.

We have no good candidates for linear formulae in elementary logic extended by quantifier “most \( x \) such that \( \varphi(x) \) satisfies \( \psi(x) \)”.

Our only linear candidates are not equivalent sentences:
Having no adequate linear formulae as logical form of (10), we must assign to (10) the following branch reading:

\[
\begin{align*}
\text{(13)} & \quad \text{Most } x \ V(x) \\
& \quad \text{Most } y \ T(y) \\
& \quad \text{[H(x, y)]},
\end{align*}
\]

We know that (13) is not equivalent to any formulae of elementary logic extended by quantifier Most (see: [11]). Moreover “empirical experience” shows that people readily interpret (10) as true in “branching models”. It would be interesting from a psychological and philosophical point of view to estimate the computational complexity for “model-checking” algorithm for (13) in a given finite models. Plausibly we need more than polynomial time [see:12]. Research on algorithmic complexity of some natural language construction can throw some light onto the matter of the structure of our linguistics abilities.

4. References


[9] MOSTOWSKI M. & MAJSK D. Computational Complexity of Semantics of Some Natural Language Constructions, IN ???.

